Basic notations:

Sectors denoted by *j*

Households denoted by h

Factors denoted by *L* and *K* and are owned by households

 Q_j is the production of goods by the *j*th sector P_j is the price of goods produced by the *j*th sector $a_{j1,j}$ is the amount of goods used from sector *j*1 when producing one unit of goods in the *j*th sector

$$L_j$$
 is the usage of labor by the j^{th} sector W_L is the price of labor (the wage rate) K_j is the usage of capital in the j^{th} sector W_K is the price of capital

Including Demand for Products and Factors

- Factor demand derived from CES function
 - : Production function

$$Q_j = \phi_j (\delta_j L_j^{(\sigma_j - 1)/\sigma_j} + (1 - \delta_j) K_j^{(\sigma_j - 1)/\sigma_j})^{\sigma_j/(\sigma_j - 1)}$$

: Factor demand

$$L_{j} = \frac{1}{\phi_{j}} Q_{j} \left[\delta_{j} + (1 - \delta_{j}) \left(\frac{\delta_{j} W_{K}}{(1 - \delta_{j}) W_{L}} \right)^{(1 - \sigma_{j})} \right]^{\sigma_{j}/(1 - \sigma_{j})}$$
$$K_{j} = \frac{1}{\phi_{j}} Q_{j} \left[\delta_{j} \left(\frac{(1 - \delta_{j}) W_{L}}{\delta_{j} W_{K}} \right)^{(1 - \sigma_{j})} + (1 - \delta_{j}) \right]^{\sigma_{j}/(1 - \sigma_{j})}$$

Note that: $\sigma \rightarrow 0$ then CES tends to Leontief $\sigma \rightarrow 1$ then CES tends to Cobb - Douglas

Including Demand for Products and factors

Household product demand from CES Utility function

Maximize Utility

$$U = \left[\sum_{j} \left(\alpha \right)^{1/\sigma} \left(X_{j}\right)^{(\sigma - 1)/\sigma}\right]^{\sigma/(\sigma - 1)}$$

s.t

$$\sum_{j} P_{j} X_{j} \leq W_{L} \overline{L} + W_{K} \overline{K} \equiv Income$$

Yields Demand Curve

$$X_{j} = \frac{\alpha_{j}(Income)}{P_{j}^{\sigma} \sum_{j} \left(\alpha_{j} \left(P_{j} \right)^{1-\sigma} \right)}$$

1. Supply-Demand identities for factors & products

a. Factor market:

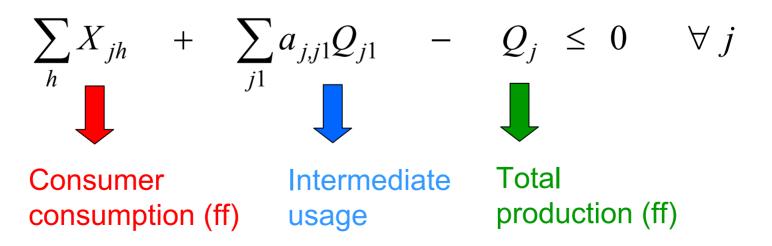
Total demand is less than or equal to total supply in every factor market or the excess demand in the factor market is less than or equal to zero.

$$\sum_{j} L_{j} - \sum_{h} \overline{L}_{h} \le 0$$
$$\sum_{j} K_{j} - \sum_{h} \overline{K}_{h} \le 0$$

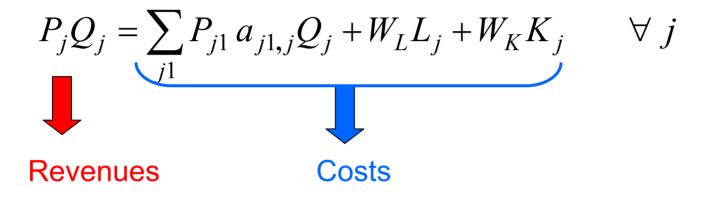
Total supply is the sum across the household endowments

- 1. Supply-Demand identities (con't)
 - b. Product or output market:

Total demand in every output market including consumer and intermediate production usage is less than or equal to total supply in that market or the excess demand in each output market is less than or equal to zero.



2. Zero profits in each sector



Note that: CRS + Perfect Competition Unit price = unit cost

3. Household income identity

This relationship implies that household exhausts its income.

$$Income_h \ge W_L \overline{L}_h + W_K \overline{K}_h$$

Note that:

Household consumptions (X_j) are a function of price and income.

Maximize U(X_j) s.t. $\sum_{j} P_{j}X_{j} \leq W_{L}\overline{L} + W_{K}\overline{K} = Income$

Recall:

Walras' Law: For any price vector **P**, **PZ(P)** = 0; i.e., the value of the excess demand, is identically zero, (Varian, page 317)

Namely, if total **demand** is **less than total supply** for the factor/commodity markets then the **price** in that market must be **zero**; otherwise, prices will be nonzero only if supply equals demand

This implies PRICE OR EXCESS DEMAND = 0. This leads to the following complementary relationships.

Complementarity (con't)

In a CGE model, a set of prices *P* and quantities *Q* are defined as variables such that D = S (**Walras' Law**)

(Qs-Qd)P = 0(Pd-P)Qd = 0(Ps-P)Qs = 0

Implications:

Each equation must be binding or an associated complementary variable must be zero.

IF P > 0 then Qs = Qd IF Qd > 0 then Pd = P, IF Qs > 0 then Ps = P, and

This is similar to KT conditions of the following optimization model.

Complementarity (con't)

1.
$$0 \le W_L$$

 $0 \le W_K$
 $1 \ge \sum_{j} L_j - \sum_{h} \overline{L}_h \le 0$
 $1 \ge W_K$
 $1 \ge \sum_{j} K_j - \sum_{h} \overline{K}_h \le 0$
representing complementary relationship

Factor prices must be zero if factors are not all used up. Non zero prices exist if factors all are consumed.

2.
$$0 \le P_j \perp \sum_h X_{jh} + \sum_{j1} a_{j,j1} Q_{j1} - Q_j \le 0 \quad \forall j$$

Product prices must be zero if products are not all consumed. Non zero prices exist if products all are consumed.

Complementarity (con't)

3.
$$0 \le Q_j \perp P_j Q_j \le \sum_{j1} P_{j1} \mathbf{a}_{j1,j} Q_j + W_L L_j + W_K K_j$$

Firm profits must equal zero and a non-zero production level is achieved.

Firm profits can be less than costs without the firm producing.

$$4. \quad 0 \leq Income_h \quad \perp \ Income_h \geq W_L \overline{L}_h + W_K \overline{K}_h$$

Household incomes must be non-zero if expenditures exhaust incomes.

Numerical Example

Symbol	Brief Description
σ_h	Elasticity of substitution in household CES
$\frac{\alpha_{jh}}{L_{h},K_{h}}$	Consumption share in household CES
$\overline{L}_{h}, \overline{K}_{h}$	Household endowments of factors
ajı, j	Use of goods in sector1 when producing in sector j
ϕ_{j}	Scale parameter in CES production function
$\phi_j \\ \delta_j \\ \sigma_j^j$	Distribution parameter in CES production
σ_{j}	Elasticity of production factor substitution
s _h	Household share of tax disbursements
^s j	Government goods purchase dependence on revenues
t _h	Household tax level
t _{fj}	Tax on factor <i>f</i> in sector <i>j</i>
F _h	Household tax exemptions

Parameter Specification

Example of simple 2x2x2 CGE (Shoven and Whalley 1984)

Production Parameters						
Sector (j)	ϕ_{j}	δ_{j}	σ_{j}			
Food	1.5	0.6	2.0			
Non-Food	2.0	0.7	0.5			

t	j
Labor	Capital
0.0	0.0
0.0	0.0

Consumer Parameters

Household (h)	σ_{h}	sh	t _h	F _h	
Farmer Non-Farmer	0.75 1.5	0.6 0.4	$\begin{array}{c} 0.00\\ 0.00\end{array}$	0.0 0.0	
	α_{hj}		Endow	ments	
	Food	Non	-Food	Labor	Capital
Farmer	0.3	0.	7	60	0

Government

Food	0.0
Non-Food	0.0

Equilibrium Results

1. Total demand for each output exactly matches the amount produced

1

	323	PARAMETER	HHdemand	Total	quantity	demand	for	output
		Food	NonFo	od				
NonFarm	ner	11.515	16.6 37.7	75		_ C	$\alpha_j(In$	$(come_h)$
Farmer Total		13.428 24.942		78		$ih = \overline{P_j^{\sigma}}$	$\sum \left(\frac{1}{2} \right)$	$\frac{1}{\alpha_{j}(P_{j})^{l-\sigma}}$
	323	PARAMETER	ProdQ To				j	
Food	24.9	942, Noi	nFood 54.3	78				
Q_j	$=\phi_j(\phi)$	$\delta_j L_j^{(\sigma_j-1)/\sigma_j}$	$+(1-\delta_j)K_j$	$(\sigma_j - 1)/\sigma_j$	$)^{\sigma_j/(\sigma_j-1)}$			
Reca	all:	$\sum_h X_{jh} +$	$-\sum_{j1}a_{j}$	2_{j1} –	$Q_j \leq 0$) ∀.	j	35

Equilibrium Results (con't)

2. Producer revenues equal consumer expenditures

-		323	PARAMETER	ConExpens	se C	onsumer	expenditures
			Food	NonF	ood		
Γ	NonFarme Parmer	er	16.110 18.787	18.: 41.:			$\sum P V$
	lotal		34.897	59.			$\sum_{h} P_{j} X_{jh}$
-		323	PARAMETER	ProdRev	Prod	ucer rev	renues
E	?ood	34.8	897, Noi	nFood 59.	439,	Total	. 94.337
			$P_j Q_j$				

3. Labor and capital are exhausted

	323	PARAMETER	ResrcQuan	Fact	cor	endowment	levels
		Food	NonFood		To	tal	
Labor Capital		26.366 6.212	33.634 18.788			000 000	

$$L_{j} = \frac{1}{\phi_{j}} Q_{j} \left[\delta_{j} + (1 - \delta_{j}) \left(\frac{\delta_{j} W_{K}}{(1 - \delta_{j}) W_{L}} \right)^{(1 - \sigma_{j})} \right]^{\sigma_{j}/(1 - \sigma_{j})}$$

$$K_{j} = \frac{1}{\phi_{j}} Q_{j} \left[\delta_{j} \left(\frac{(1 - \delta_{j}) W_{L}}{\delta_{j} W_{K}} \right)^{(1 - \sigma_{j})} + (1 - \delta_{j}) \right]^{\sigma_{j}/(1 - \sigma_{j})}$$

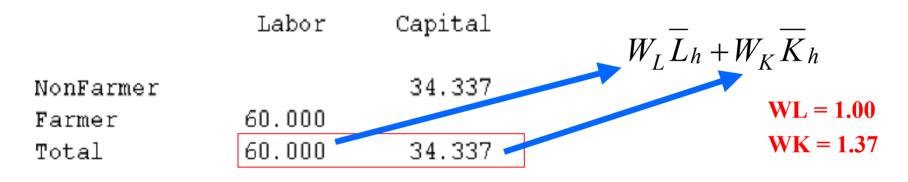
4. Unit cost = selling price => zero profits

	323 PAR	AMETER Unit	Cost	Unit cost	
Food	1.399,	NonFood	1.093		wl + rk
	323 PAR	AMETER Unit	Price	Unit selli	ng price.
Food	1.399,	NonFood	1.093		From model solutions

Equilibrium Results (con't)

5. Consumer factor incomes equal producer factor costs

---- 323 PARAMETER ResrcIncom Consumer factor incomes



---- 323 PARAMETER ResrcCost Producer factor costs

Food NonFood Total Labor 26.366 33.634 60.000 Capital 8.532 25.805 34.337 $W_LL_i + W_KK_i$

Equilibrium Results (con't)

6. Household expenditures exhaust their incomes

	323	PARAMETER	ConExpense	Consumer expenditures
		Food	NonFood	
NonFarm Farmer Total	er	16.110 18.787 34.897	18.227 41.213 59.439	$\sum_{j} P_{j} X_{j}$
	323	PARAMETER	ResrcIncom	Consumer factor incomes
		Labor	Capital	
NonFarm Farmer	er	60.000	34.337	$\implies W_L \overline{L}_h + W_K \overline{K}_h$
Total		60.000	34.337	

Inconsistent Data

Because calibration relies on the benchmark data, what to do if

- **:** Data/Accounting inconsistency
 - \Rightarrow demand \neq supply
 - => expenditures exceed incomes
 - => consumer expenditure classification does not match production classification
 - => lack of data

DATA PROCESSING & ADJUSTMENT! => No uniform adjustment

- => adjustment varies from case to case
- => interpolation and use of other economic data
- => use previous year data with some adjustment
- => RAS (row-and-column-sum) procedure
- => modeler's judgment

Suggested Reading: St-Hilaire, F., and J. Whalley. "A microconsistent equilibrium data set for Canada for use in tax policy analysis." Review of Income and Wealth 29, 175-204.

Building the Basic Data – things to do

Things to be considered when building the basic data

- 1. Check the classifications among data sets
 - e.g. HH expenditures categories vs. industry product categories
- 2. Decide on units for goods and factors so that prices and quantities are separately obtained
 - e.g. choose units for goods and factors so that they have a price of unity in the benchmark equilibrium
 - Note: in the CGE model only the **<u>relative price</u>** is the focus and the absolute price is not important.

Building the Basic Data – things to do

- 3. Check if the data is consistent with the equilibrium conditions e.g.
 - a. Demands = Supplies (consumption = production)
 - b. Zero profits (revenues = costs)
 - c. All agents (i.e. HH, Government, ROW) exhaust their budgets
 - d. Resources are used up.

Suggested Reading: St-Hilaire, F., and J. Whalley. "A microconsistent equilibrium data set for Canada for use in tax policy analysis." Review of Income and Wealth 29, 175-204.