Fundamental Structure

- Basic notations:

Sectors denoted by j

Households denoted by h

Factors denoted by L and K and are owned by households

Q_j is the production of goods by the j^{th} sector

P_j is the price of goods produced by the j^{th} sector

$a_{j1,j}$ is the amount of goods used from sector $j1$ when producing one unit of goods in the j^{th} sector

L_j is the usage of labor by the j^{th} sector

W_L is the price of labor (the wage rate)

K_j is the usage of capital in the j^{th} sector

W_K is the price of capital

Including Demand for Products and Factors

- Factor demand derived from CES function

: Production function

$$Q_j = \phi_j(\delta_j L_j^{(\sigma_j - 1)/\sigma_j} + (1-\delta_j)K_j^{(\sigma_j - 1)/\sigma_j})^{\sigma_j/(\sigma_j - 1)}$$

: Factor demand

$$L_j = \frac{1}{\phi_j} Q_j \left[\delta_j + (1-\delta_j)\left(\frac{\delta_j W_K}{(1-\delta_j)W_L} \right)^{(1-\sigma_j)} \right]^{\sigma_j/(1-\sigma_j)}$$

$$K_j = \frac{1}{\phi_j} Q_j \left[\delta_j \left(\frac{(1-\delta_j)W_L}{\delta_j W_K} \right)^{(1-\sigma_j)} + (1-\delta_j) \right]^{\sigma_j/(1-\sigma_j)}$$

Note that: $\sigma \to 0$ then CES tends to Leontief

$\sigma \to 1$ then CES tends to Cobb - Douglas

Including Demand for Products and factors

- Household product demand from CES Utility function

Maximize Utility

$$U = \left[\sum_j (\alpha)^{1/\sigma} (X_j)^{(\sigma - 1)/\sigma} \right]^{\sigma/(\sigma - 1)}$$

s.t

$$\sum_j P_j X_j \le W_L \overline{L} + W_K \overline{K} \equiv Income$$

Yields Demand Curve

$$X_j = \frac{\alpha_j (Income)}{P_j^{\sigma} \sum_j \left(\alpha_j (P_j)^{1-\sigma} \right)}$$

Fundamental Structure

1. Supply-Demand identities for factors & products

a. Factor market:

Total demand is less than or equal to total supply in every factor market or the excess demand in the factor market is less than or equal to zero.

$$\sum_j L_j - \sum_h \overline{L}_h \leq 0$$

$$\sum_j K_j - \sum_h \overline{K}_h \leq 0$$

Total supply is the sum across the household endowments

Fundamental Structure

1. **Supply-Demand identities (con't)**

   b. Product or output market:

   Total demand in every output market including consumer and intermediate production usage is less than or equal to total supply in that market or the excess demand in each output market is less than or equal to zero.

$$\sum_{h} X_{jh} \quad + \quad \sum_{j1} a_{j,j1} Q_{j1} \quad - \quad Q_j \;\leq\; 0 \qquad \forall\, j$$

Consumer consumption (ff)

Intermediate usage

Total production (ff)

Fundamental Structure

2. Zero profits in each sector

$$\underbrace{P_j Q_j}_{\text{Revenues}} = \underbrace{\sum_{j1} P_{j1}\, a_{j1,j} Q_j + W_L L_j + W_K K_j}_{\text{Costs}} \qquad \forall\ j$$

Note that: CRS + Perfect Competition

Unit price = unit cost

Fundamental Structure

3. Household income identity

This relationship implies that household exhausts its income.

$$Income_h \geq W_L \overline{L}_h + W_K \overline{K}_h$$

Note that:

Household consumptions (X_j) are a function of price and income.

Maximize $U(X_j)$

s.t. $\sum_j P_j X_j \leq W_L \overline{L} + W_K \overline{K} = Income$

Complementarity

Recall:

Walras' Law: *For any price vector* ***P***, ***PZ(P)*** *= 0; i.e., the value of the excess demand*, *is identically zero,* (Varian, page 317)

Namely, if total **demand** is less than total **supply** for the factor/commodity markets then the **price** in that market must be zero; otherwise, prices will be nonzero only if supply equals demand

This implies PRICE OR EXCESS DEMAND = 0.

This leads to the following complementary relationships.

Complementarity (con't)

In a CGE model, a set of prices *P* and quantities *Q* are defined as variables such that D = S (**Walras' Law**)

$$(Qs-Qd)\boldsymbol{P} = 0$$

$$(Pd-P)\boldsymbol{Qd} = 0$$

$$(Ps-P)\boldsymbol{Qs} = 0$$

Implications:

Each equation must be binding or an associated complementary variable must be zero.

IF P > 0 then Qs = Qd

IF Qd > 0 then Pd = P,

IF Qs > 0 then Ps = P, and

This is similar to KT conditions of the following optimization model.

Complementarity (con't)

1.

$$0 \le W_L \quad \perp \sum_j L_j - \sum_h \bar{L}_h \le 0$$

$$0 \le W_K \quad \perp \sum_j K_j - \sum_h \bar{K}_h \le 0$$

representing complementary relationship

Factor prices must be zero if factors are not all used up.

Non zero prices exist if factors all are consumed.

2. $$0 \le P_j \quad \perp \sum_h X_{jh} + \sum_{j1} a_{j,j1} Q_{j1} - Q_j \le 0 \qquad \forall j$$

Product prices must be zero if products are not all consumed.
Non zero prices exist if products all are consumed.

Complementarity (con't)

3. $$0 \leq Q_j \quad \perp \quad P_j Q_j \leq \sum_{j1} P_{j1} \mathbf{a_{j1,j}} Q_j + W_L L_j + W_K K_j$$

Firm profits must equal zero and a non-zero production level is achieved.

Firm profits can be less than costs without the firm producing.

4. $$0 \leq Income_h \quad \perp \quad Income_h \geq W_L \overline{L}_h + W_K \overline{K}_h$$

Household incomes must be non-zero if expenditures exhaust incomes.

Numerical Example

Symbol	Brief Description
σ_h	Elasticity of substitution in household CES
α_{jh}	Consumption share in household CES
$\bar{L}_h, \bar{K}_h$	Household endowments of factors
$a_{j1,j}$	Use of goods in sector1 when producing in sector j
ϕ_j	Scale parameter in CES production function
δ_j	Distribution parameter in CES production
σ_j	Elasticity of production factor substitution
s_h	Household share of tax disbursements
s_j	Government goods purchase dependence on revenues
t_h	Household tax level
t_{fj}	Tax on factor *f* in sector *j*
F_h	Household tax exemptions

Parameter Specification

- **Example of simple 2x2x2 CGE (Shoven and Whalley 1984)**

Production Parameters

Sector (j)	ϕ_j	δ_j	σ_j
Food	1.5	0.6	2.0
Non-Food	2.0	0.7	0.5

t_{fj}

Labor	Capital
0.0	0.0
0.0	0.0

Consumer Parameters

Household (h)	σ_h	s_h	t_h	F_h
Farmer	0.75	0.6	0.00	0.0
Non-Farmer	1.5	0.4	0.00	0.0

	α_{hj}		Endowments	
	Food	Non-Food	**Labor**	**Capital**
Farmer	0.3	0.7	60	0
Non-Farmer	0.5	0.5	0	25

Government

Food	0.0
Non-Food	0.0

Equilibrium Results

1. **Total demand for each output exactly matches the amount produced**

```
----     323 PARAMETER HHdemand  Total quantity demand for output

                     Food      NonFood

NonFarmer          11.515       16.675
Farmer             13.428       37.704
Total              24.942       54.378


----     323 PARAMETER ProdQ  Total quantity produced

Food     24.942,     NonFood 54.378
```

$$X_{jh} = \frac{\alpha_j \left(Income_h\right)}{P_j^{\sigma} \sum_j \left(\alpha_j \left(P_j\right)^{1-\sigma}\right)}$$

$$Q_j = \phi_j \left(\delta_j L_j^{(\sigma_j - 1)/\sigma_j} + (1-\delta_j) K_j^{(\sigma_j - 1)/\sigma_j}\right)^{\sigma_j/(\sigma_j - 1)}$$

Recall: $\sum_h X_{jh} + \sum_{j1} \cancel{a_{jj1} Q_{j1}} - Q_j \le 0 \quad \forall j$

Equilibrium Results (con't)

2. Producer revenues equal consumer expenditures

```
----     323 PARAMETER ConExpense  Consumer expenditures

                Food      NonFood

NonFarmer      16.110      18.227
Farmer         18.787      41.213
Total          34.897      59.439
```

$\sum_h P_j X_{jh}$

```
----     323 PARAMETER ProdRev  Producer revenues

Food     34.897,    NonFood 59.439,    Total    94.337
```

$P_j Q_j$

Equilibrium Results (con't)

3. Labor and capital are exhausted

```
----     323 PARAMETER ResrcQuan  Factor endowment levels

                  Food      NonFood        Total

Labor           26.366       33.634       60.000
Capital          6.212       18.788       25.000
```

$$L_j = \frac{1}{\phi_j} Q_j \left[\delta_j + (1-\delta_j) \left(\frac{\delta_j W_K}{(1-\delta_j) W_L} \right)^{(1-\sigma_j)} \right]^{\sigma_j/(1-\sigma_j)}$$

$$K_j = \frac{1}{\phi_j} Q_j \left[\delta_j \left(\frac{(1-\delta_j) W_L}{\delta_j W_K} \right)^{(1-\sigma_j)} + (1-\delta_j) \right]^{\sigma_j/(1-\sigma_j)}$$

Equilibrium Results (con't)

4. Unit cost = selling price => zero profits

```
----      323 PARAMETER UnitCost  Unit cost

Food      1.399,      NonFood 1.093
```

$wl + rk$

```
----      323 PARAMETER UnitPrice  Unit selling price

Food      1.399,      NonFood 1.093
```

From model solutions

Equilibrium Results (con't)

5. **Consumer factor incomes equal producer factor costs**

```
----     323 PARAMETER ResrcIncom  Consumer factor incomes

                Labor     Capital

NonFarmer                  34.337
Farmer         60.000
Total          60.000      34.337
```

$$W_L \overline{L}_h + W_K \overline{K}_h$$

WL = 1.00

WK = 1.37

```
----     323 PARAMETER ResrcCost  Producer factor costs

               Food     NonFood       Total

Labor        26.366      33.634      60.000
Capital       8.532      25.805      34.337
```

$$W_L L_j + W_K K_j$$

Equilibrium Results (con't)

6. Household expenditures exhaust their incomes

```
----     323 PARAMETER ConExpense  Consumer expenditures

                   Food      NonFood

NonFarmer        16.110       18.227
Farmer           18.787       41.213
Total            34.897       59.439
```

$$\rightarrow \sum_j P_j X_j$$

```
----     323 PARAMETER ResrcIncom  Consumer factor incomes

                  Labor      Capital

NonFarmer                     34.337
Farmer           60.000
Total            60.000       34.337
```

$$\rightarrow W_L \overline{L}_h + W_K \overline{K}_h$$

Inconsistent Data

Because calibration relies on the benchmark data, what to do if

: Data/Accounting inconsistency

=> demand $\neq$ supply

=> expenditures exceed incomes

=> consumer expenditure classification does not match production classification

=> lack of data

DATA PROCESSING & ADJUSTMENT! => No uniform adjustment

=> adjustment varies from case to case

=> interpolation and use of other economic data

=> use previous year data with some adjustment

=> RAS (row-and-column-sum) procedure

=> modeler's judgment

Suggested Reading: St-Hilaire, F., and J. Whalley. "A microconsistent equilibrium data set for Canada for use in tax policy analysis." Review of Income and Wealth 29, 175-204.

Building the Basic Data – things to do

Things to be considered when building the basic data

1. Check the classifications among data sets

   e.g. HH expenditures categories vs. industry product categories

2. Decide on units for goods and factors so that prices and quantities are separately obtained

   e.g. choose units for goods and factors so that they have a price of unity in the benchmark equilibrium

   Note: in the CGE model only the **relative price** is the focus and the absolute price is not important.

Building the Basic Data – things to do

3. Check if the data is consistent with the equilibrium conditions e.g.

    a. Demands = Supplies (consumption = production)

    b. Zero profits (revenues = costs)

    c. All agents (i.e. HH, Government, ROW) exhaust their budgets

    d. Resources are used up.

Suggested Reading:
St-Hilaire, F., and J. Whalley. "A microconsistent equilibrium data set for Canada for use in tax policy analysis." Review of Income and Wealth 29, 175-204.